Quantification of the role of non-linear quadruplet wave-wave interactions in wave models

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Some history

Directional response of waves on turning winds

Hasselmann & Hasselmann

Visiting Ian Young in 1989 at ADFA, Canberra

Trying to understand the workings of the non-linear interactions and its methods with focus on the DIA and WRT method

Visit to Bureau of Meteorology here in Melbourne presenting the WAM model. Took a number of years before it really started

Young, I.R., and G.Ph. Van Vledder, 1993. The central role of nonlinear four-wave interactions in wind-wave modelling. Proc. Roy Soc. London

Focus

Wind generated ocean waves

Spectral Modelling using 3G-wave prediction models, description by Wave Action Balance Equation, where $N = N(\sigma, \theta, x, y, t)$

$$\frac{\partial}{\partial t} N + \frac{\partial}{\partial x}(c_x N) + \frac{\partial}{\partial y}(c_y N) + \frac{\partial}{\partial \theta}(c_\theta N) + \frac{\partial}{\partial \sigma}(c_\sigma N) = \boldsymbol{S}$$

Source terms describe input, dissipation and redistribution of wave action/energy in spectrum

$$S = S_{inp} + S_{wcap} + \mathbf{S_{nl4}} + S_{fric} + S_{nl3} + \ldots$$

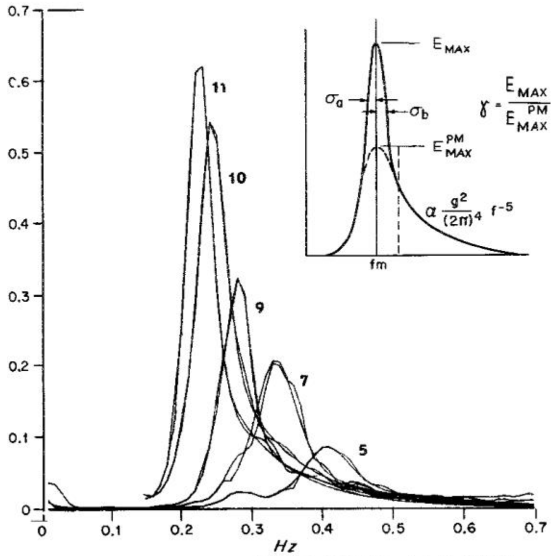
The focus here is on Snl4: the non-linear four-wave interactions (quadruplets)

Non-linear four-wave interactions

Theoretical framework by Hasselmann (1962) &Zakharov (1968)

Central role of Snl4 in wind-wave evolution became evident in JONSWAP project (Hasselmann et al, 1973)

These interactions redistribute wave energy and force spectrum to certain preferred spectral shapes and self-similar relationships (-> JONSWAP spectrum is the result)



ig. 2.5. Evolution of wave spectrum with fetch for offshore winds ($11^h - 12^h$, Sept. 15, 1968). Numbers refer to stations (cf. Fig. 1.2.). The best-fit analytical spectra (2.4.1) are also shown. The inset illustrates the definition of the five free parameters in form (2.4.1)

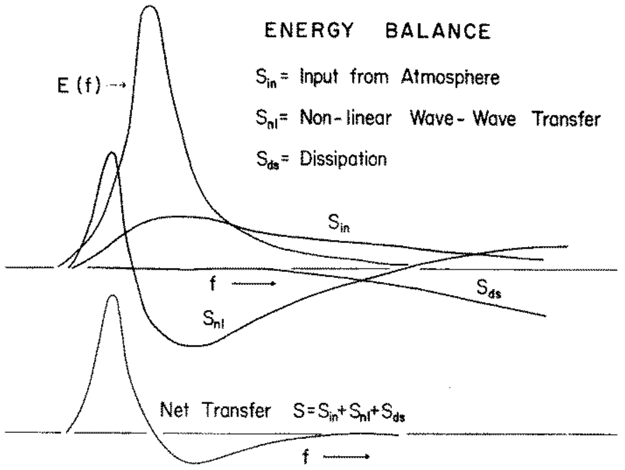
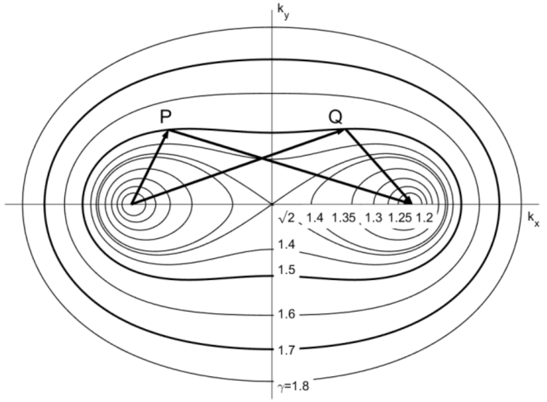


Fig. 2.22. Schematic energy balance for the case of negligible dissipation in the main part of the spectrum

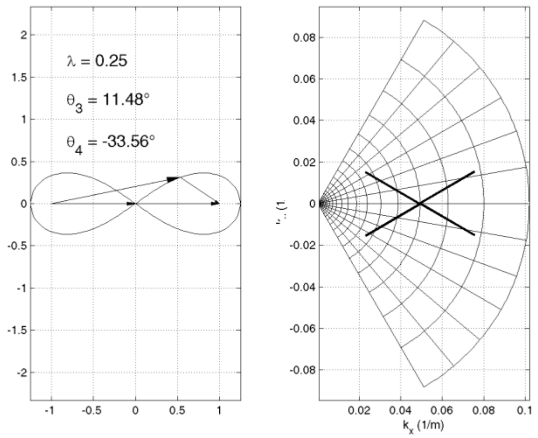
Computational methods

A closed description is available for Snl4 describing the rate of change of wave action due to all possible interactions between pairs of 4 wave number vectors:



$$\frac{\partial n_1}{\partial t} = \iiint G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \times \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4) \times \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$$
$$\times \left[n_1 n_2 (n_3 + n_4) - (n_1 + n_2) n_3 n_4 \right] d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4$$

This expression is too time consuming for application in operational wave prediction models, leading to development of Discrete Interaction Approximation (DIA) and first WAM model (WAMDIG, 1988)



Exact methods (Xnl)
continuum of interactions
Time consuming
Accurate

Discrete Interaction Approximation (DIA)
One wave number configuration
Fast
Inaccurate

Question: what do we need?

What is between Xnl and DIA ?

Quantifying the role of quadruplet interactions

$$S = S_{inp} + S_{wcap} + \mathbf{S_{nl4}} + S_{fric} + S_{nl3} + \ldots$$

$$M_S = sign(S_S) \times \int_0^{2\pi} \int_{f_{\text{low}}}^{f_{\text{high}}} |S_S(f,\theta)|\, df\, d\theta$$

Source term balance in a severe storm in the North Sea by using SWAN model simulations

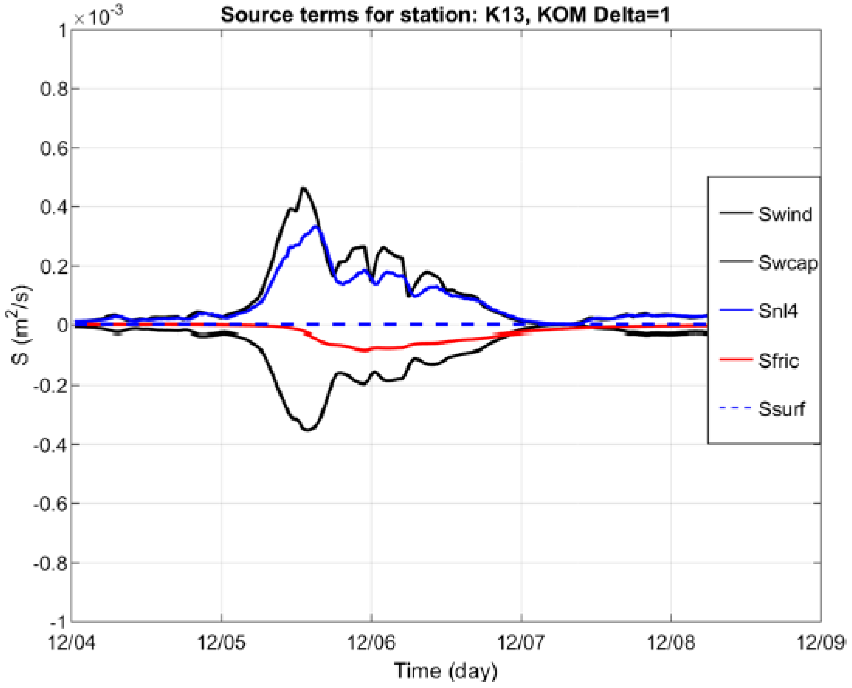
Van Vledder, Hulst, McConochie, Ocean Dynamics, 2016, Special Issue

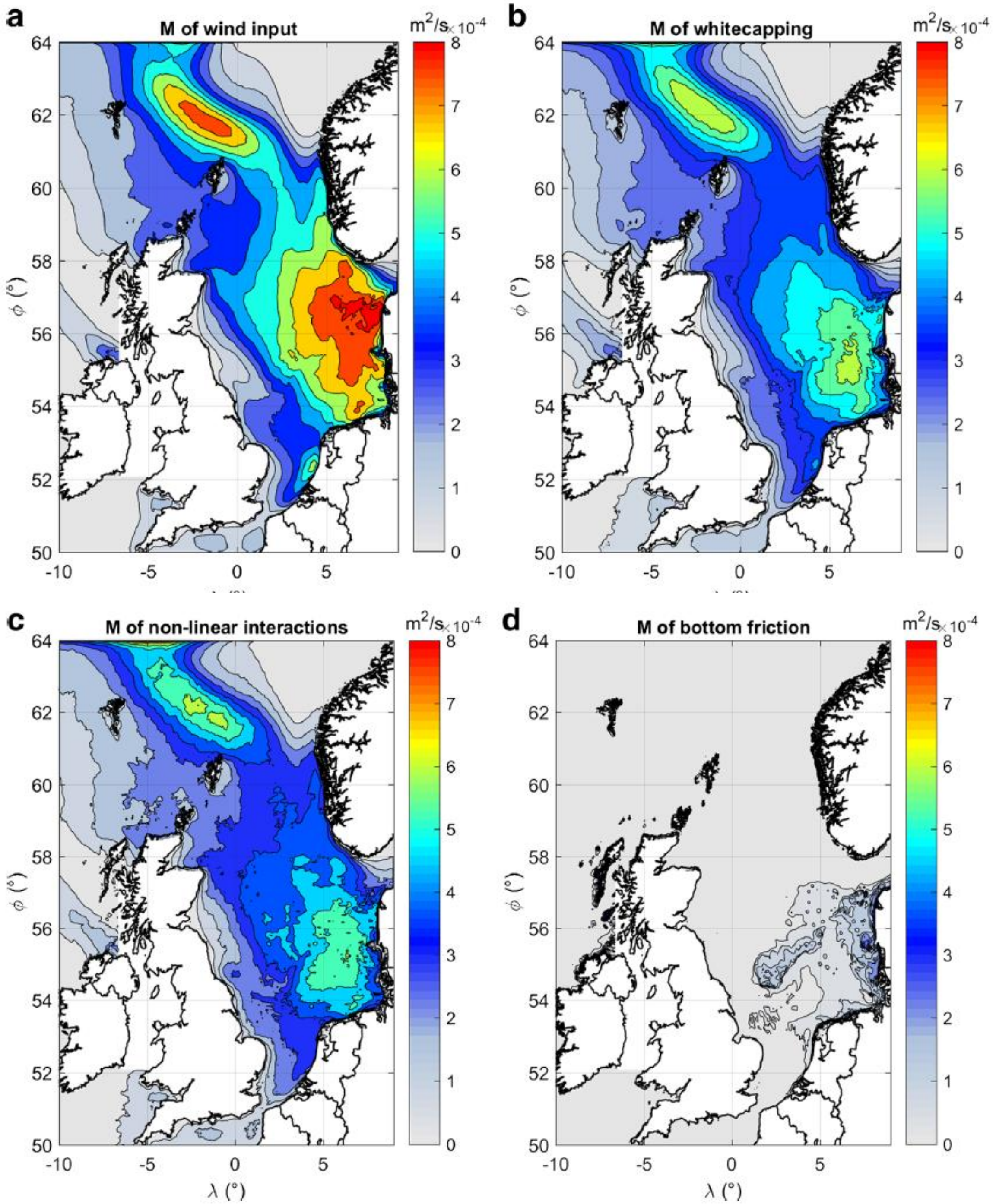
Integral magnitude

Storm of 5 December 2013

During Banff wave workshop

Snapshot at 15:00 hours



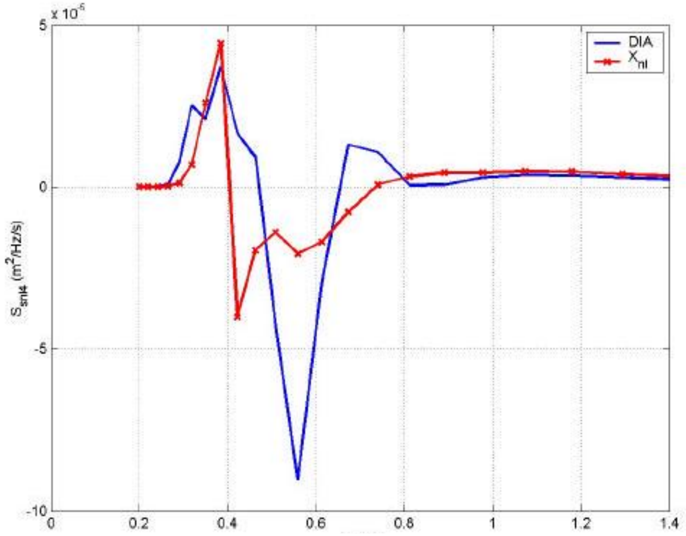


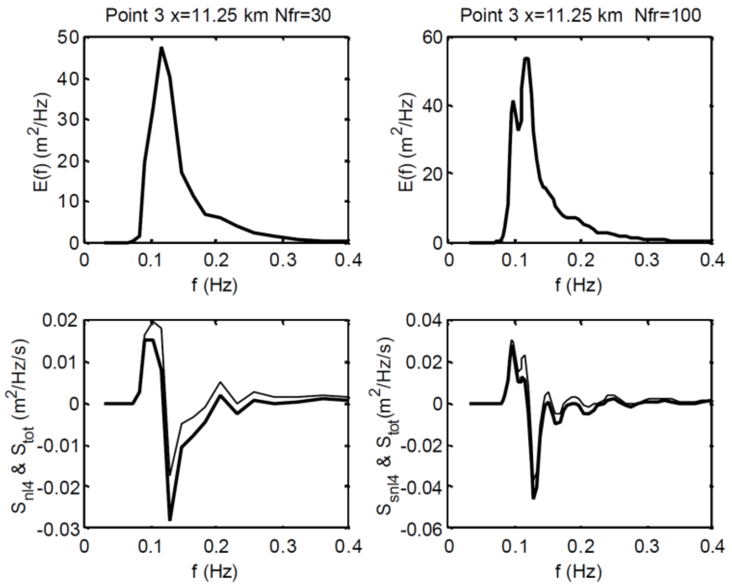
Spectral shape for academic spectra

- DIA now workhorse in 3G wave models like WAM, Wavewatch III, SWAN, and many others. Often treated as a black-box
- For academic JONSWAP spectrum large differences between Xnl and DIA
- 3G-models are tuned to compensate for deficiencies in DIA (Van Vledder et al., 2000)
- They produce reasonable/acceptable results in many situations

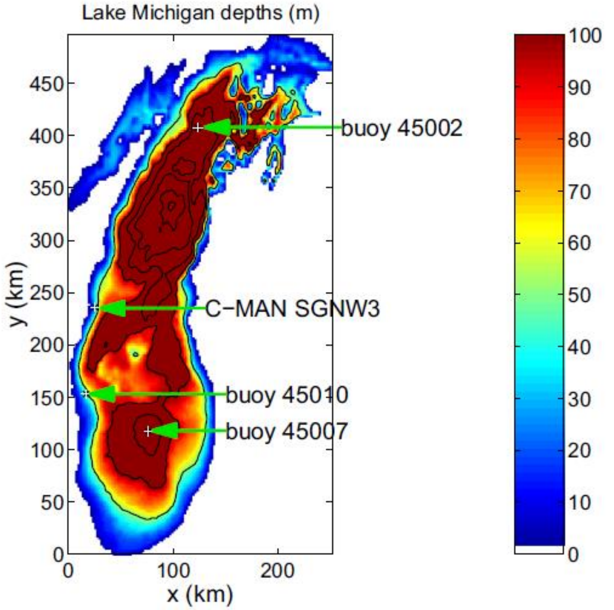
- **But what's wrong with DIA ?**
  - Mismatches with exact solution
  - Incorrect spectral shape
  - Detailed balance is often wrong (Resio, Vincent, Ardag)
  - Direction and frequency spreading too wide
  - Intrinsic properties of DIA require spacing $f_{i+1} \approx 1.1 f_i$
  - Turning wind situation in hurricanes, frontal passages
  - Effect on swell evolution and arrival times
  - Shape of spectral tail
  - ....
- What price do we want to pay for better results?

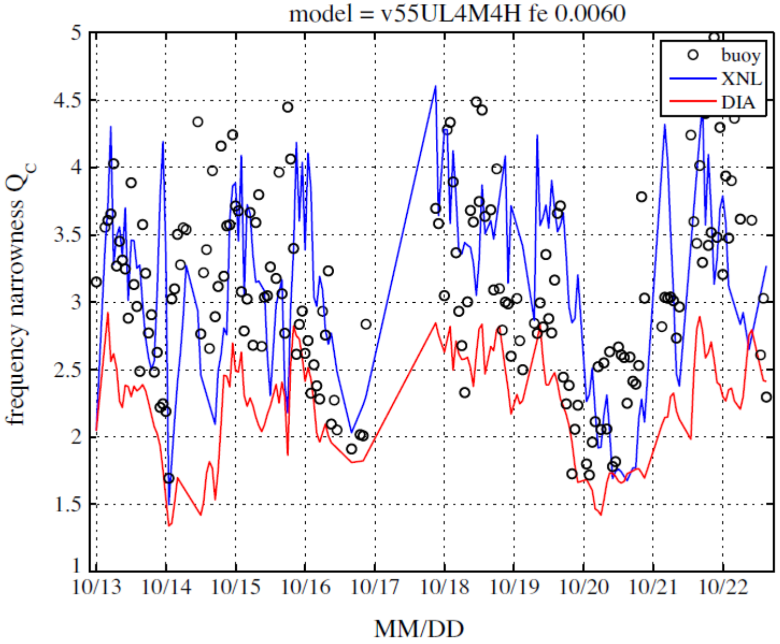
DIA & Xnl

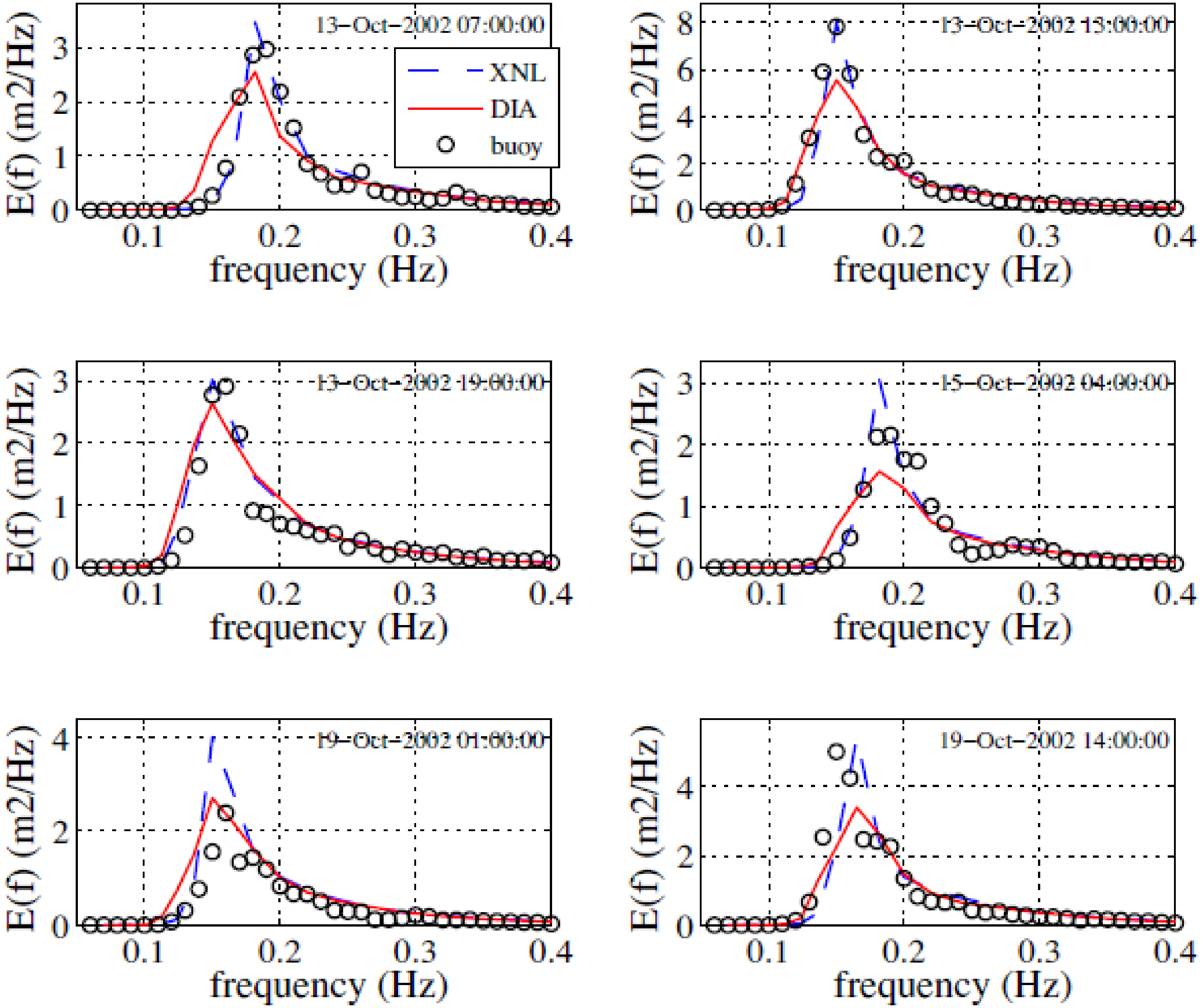




Effect of spectral width. Rogers and Van Vledder (2013)







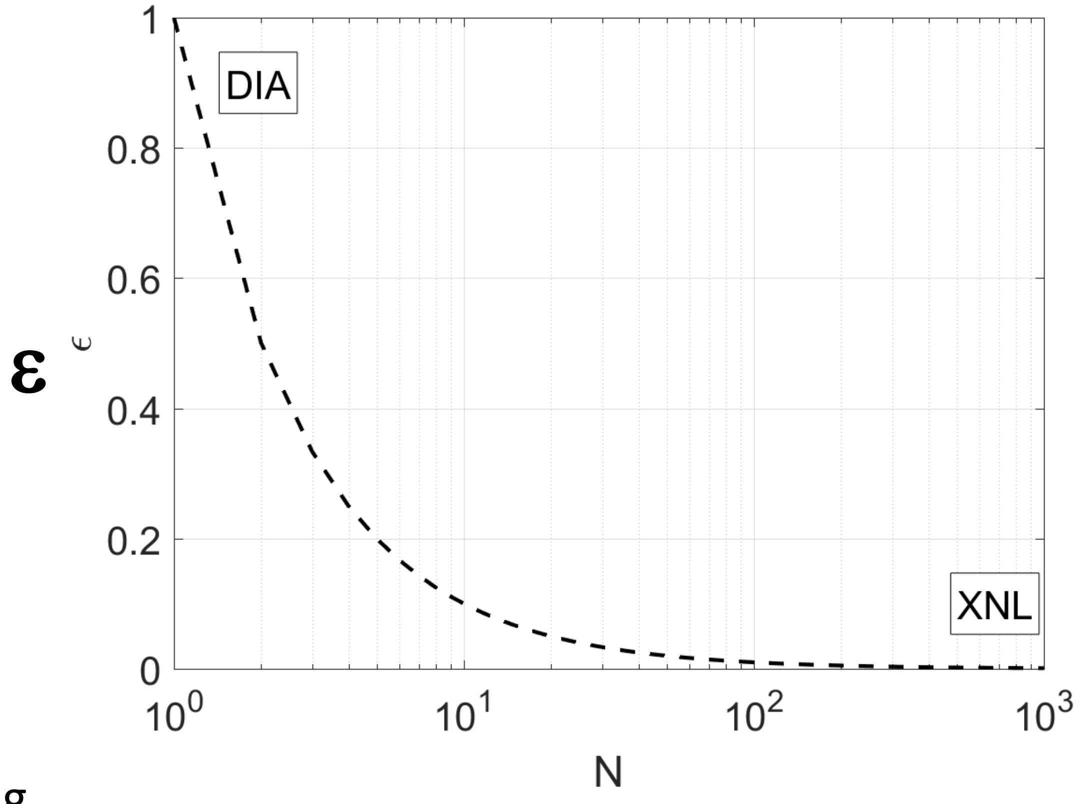
What price do we want/need to pay?

- Accuracy versus computational requirements

- Definition of accuracy (error ε)
  - of Snl4 (f,θ) compared to Xnl4(f,θ)
  - of Model Performance
    - Parameters (Hs, Tp, Tm, DIR, DSPR, Qp, κ, …)
    - 1D-spectrum E(f) or N(k)
    - Directional distribution D(θ)
    - Full 2d- spectrum E(f, θ), or N(k, θ)

- How to quantify computational requirements?
  - Focus on Snl4 term as it is the dominant term in 3G-wave model
  - Reduce to basic number of operations N providing objective criterion of workload

- Need for scalable method of computing Snl4 source term

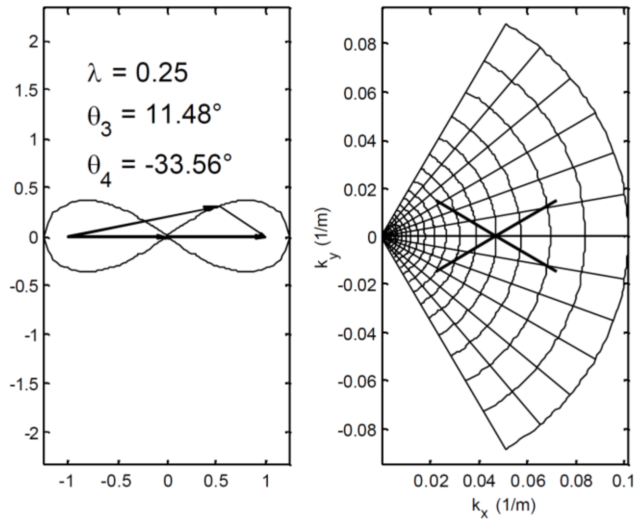


How to arrive at scalable method

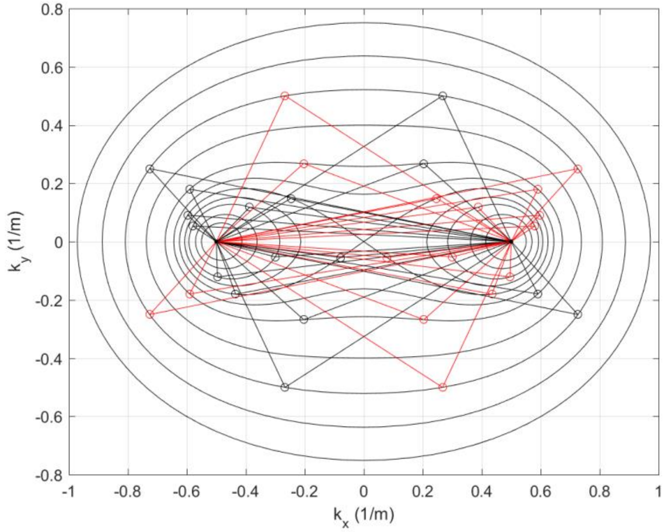
How to improve the DIA? Basically 2 methods explored.

Extend the DIA with additional wave number configurations?
- Multiple DIA's (Kawaguchi and Hashimoto ,2001; Van Vledder, 2000, 2001),
- Hasselmann, 1984: The first DIA had 2 configurations λ=0.25, λ=0.15 (added value of second configuration too limited)
- Concept of generalized multiple DIA (Van Vledder, 2001; Tolman 2004;
- **GMD (Tolman 2013)** , important result shows our goal is feasible
- xDIA (Geogjaev and Zakharov, 2018)
- Basic problem is to find next best configuration, no simple strategy exists. In **Figure of 8** many configuration are not needed

Reduce an exact method (Webb, Masuda, Lavrenov, ….)
- Reduced Interaction Approximation (Lin and Perrie, 1998)
- SRIAM (Komatsu and Masuda, 1996)
- Advanced Dominant Interaction Approximation (Perrie et al. 2010)
- Two-Scale Approximation (Resio and Perrie, 2008)



Systematic use of curves in Figure of 8



A scalable method for Snl4 to fill gap between Xnl and DIA

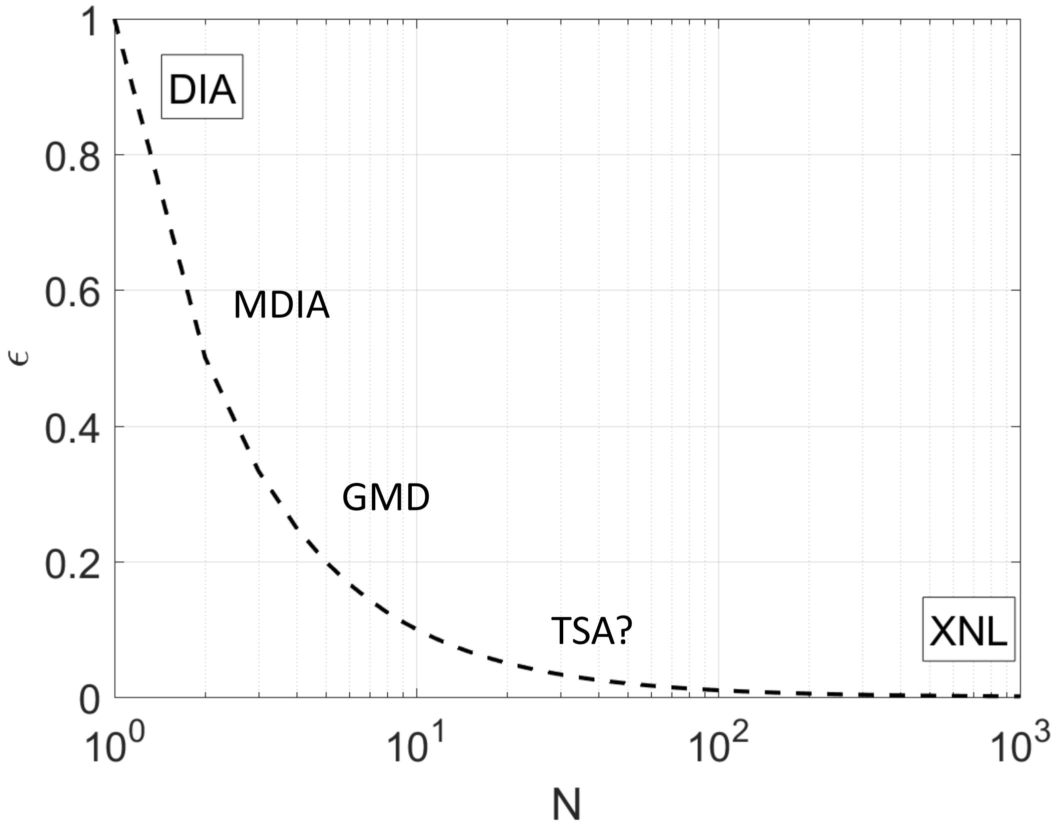
Development of a **scalable** method for the computation of non-linear four-wave interactions in a **discrete** spectral wave model based on the WRT method.

Scalable refers to the computational **workload** **N** in relation to accuracy ε, ranging from the crude DIA to the accurate XNL

Accuracy ε **may** refer to:

ε

- error in the evaluation of Snl4
- Prediction error of wave parameter (e.g. H_{m0}, T_p, θ, …).
- error in predicted spectral shape E(f), D(θ), E(f,θ) See figure (Rogers and Van Vledder, 2013) : is this improvement in spectral shape worth 1000 times more CPU?



What is good **balance** between N and operational requirement ε? … knowing that forcing, other source terms and numerics also affect quality of a wave model !

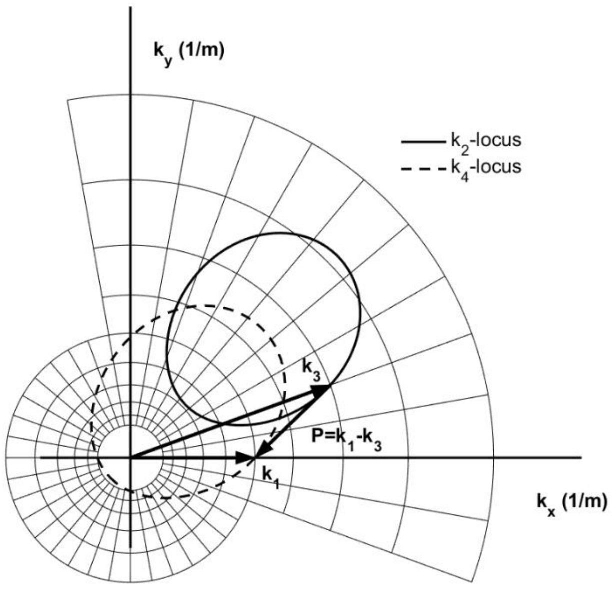
WRT method

Method based on Webb (1978) who recast Boltzmann integral in computationally feasible method as done by (Tracy and Resio, 1982) for a **discrete** wave spectrum (hence WRT)

$$S_{nl4}\left(\boldsymbol{k}_1\right)=\int T\left(\boldsymbol{k}_1,\boldsymbol{k}_3\right)d\boldsymbol{k}_3$$

with

$$T\left(\boldsymbol{k}_1,\boldsymbol{k}_3\right)=\int_s C\left(s\right)J\left(s\right)N_{1,2,3,4}\left(s\right)ds$$



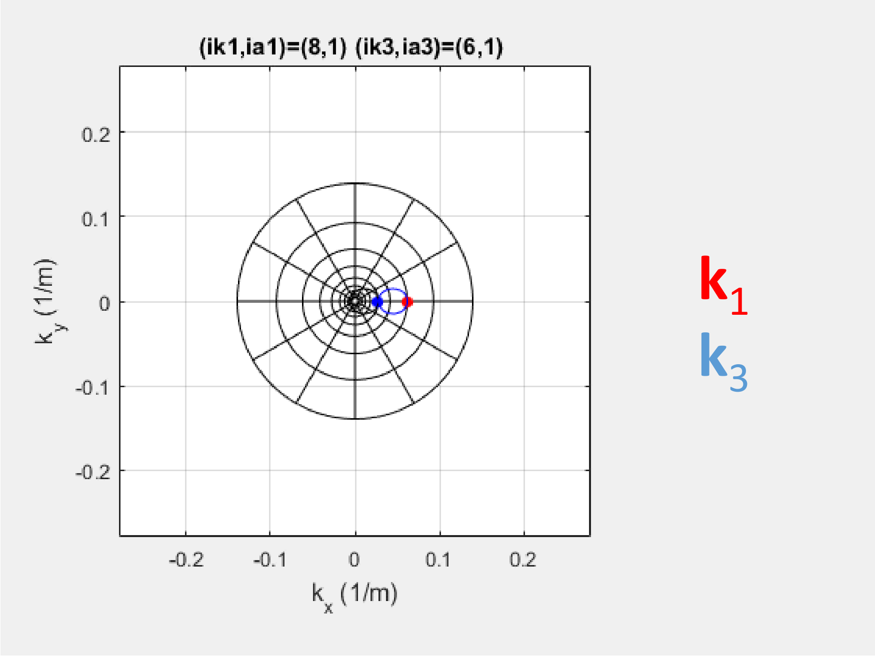
Key elements of scalable WRT method

Adaptive with respect to known part “C(s)J(s)“ of basic integrals in WRT method, where C(s) coupling coefficient and J(s) the Jacobean term

Scalable by **hierarchical** inclusion of contributions in decreasing order of pre-computed significance

Account for different **scales**

All preparations can be done in pre-processing for a certain spectral discretisation and water depth



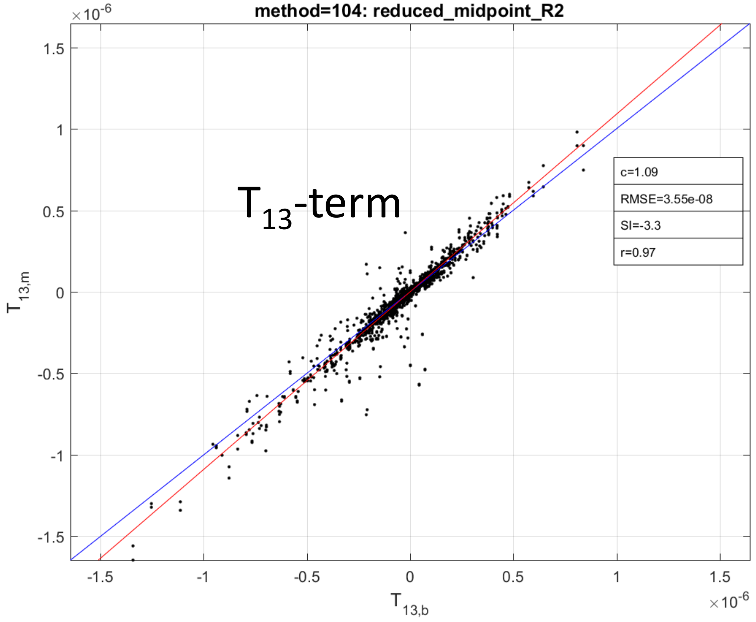
Results of optimization and scalability for academic spectra

$$S_{nl4}\left(\boldsymbol{k}_1\right) = \int T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) d\boldsymbol{k}_3$$

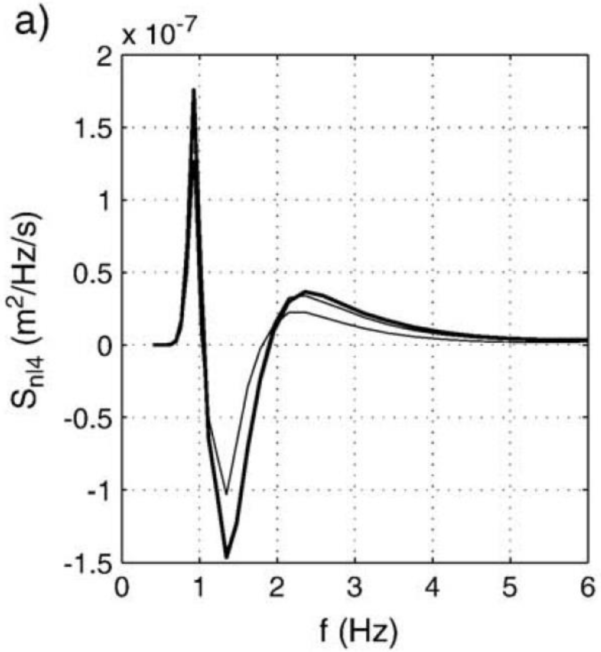
$$T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) = \int_s C\left(s\right) J\left(s\right) N_{1,2,3,4}\left(s\right) ds$$

Reductions leads to increase in error in shape of transfer rate.
This in turn affects spectral shape and spectral evolution

Effect of different step sizes on evaluation of T13 term



Validation of T13 against exact solution



Effect on Snl4(f)

Reduction of workload leads to changes in shape

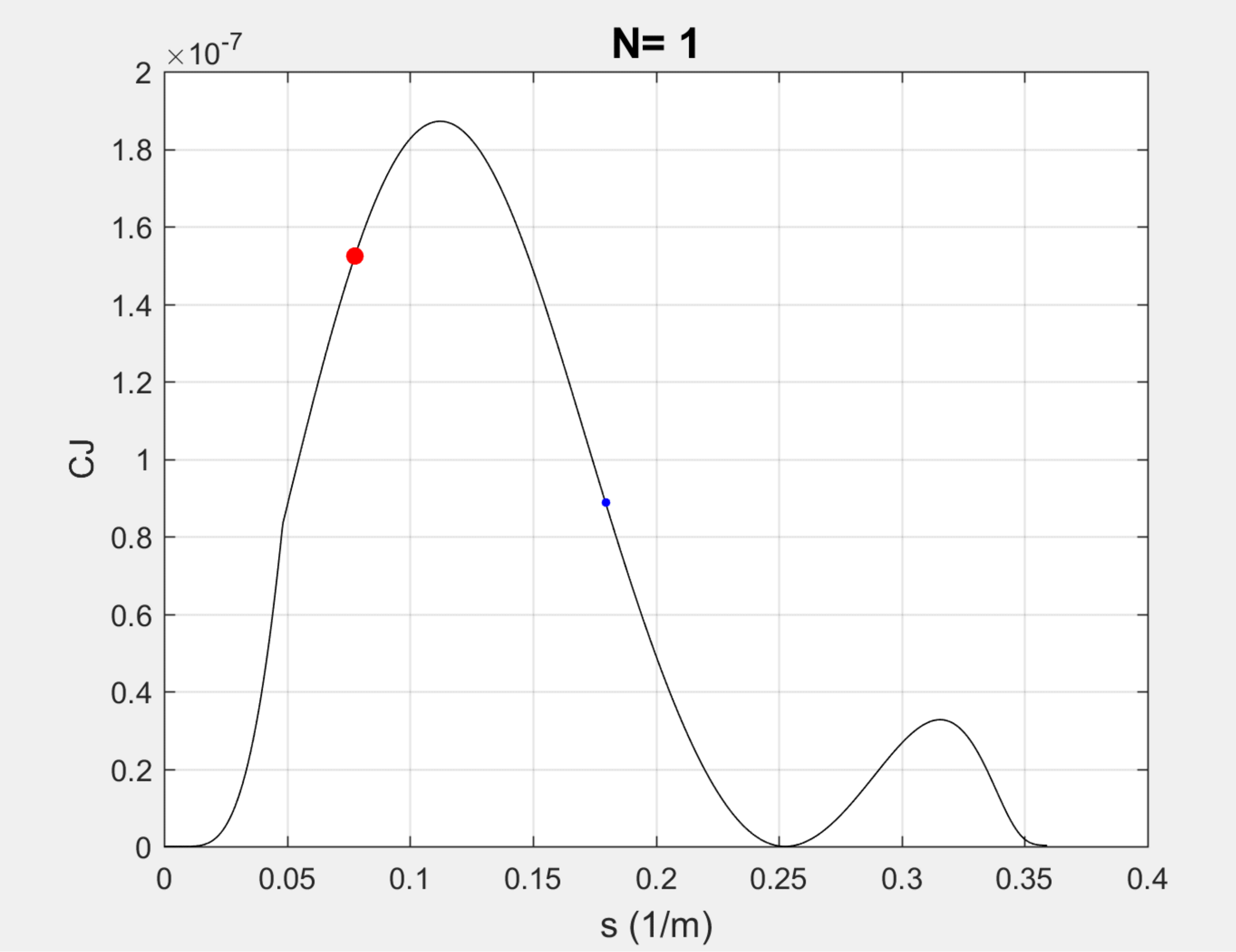
Evaluation of T13 term using lumping

Lumping of contributions of CJ in each segment around selected points on locus, (account for periodicity)

$$T_{13} \approx \sum_{i=1}^{N} N_{1,2,3,4}(s_i) \underbrace{\int_{s_i-\frac{1}{2}\Delta s_i}^{s_i+\frac{1}{2}\Delta s_i} C(s)\,J(s)\,ds}_{lumping}$$

Integrand can (again) be cast into finite sum while accounting for relation to discrete spectral grid

$$T_{13} \approx \sum_{i=1}^{N_1} v_i n_{p(i)} + \sum_{j=1}^{N_2} w_j n_{q(j)} n_{r(j)}$$



Test scalable method for dynamic situations

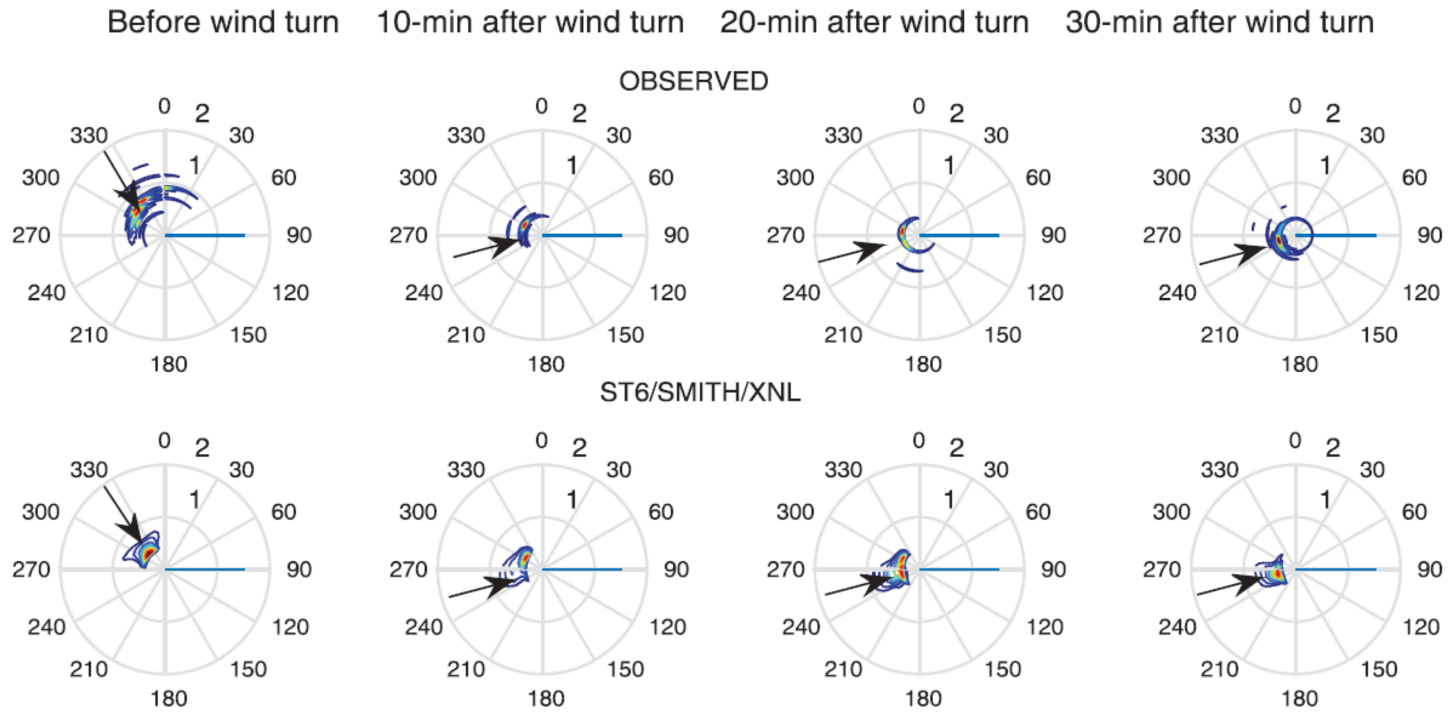
Growth curves
Directional response
Gap in spectrum
Swell propagation

Field cases
(cf. Van Vledder and Holthuijsen, 199:
Aijaz et al. 2016 (Lake George)

Determine criteria to determine when
accuracy and workload N

Spectral shape
Integral wave parameters

Implementation 3G wave models
XNLINIT and XNLMAIN

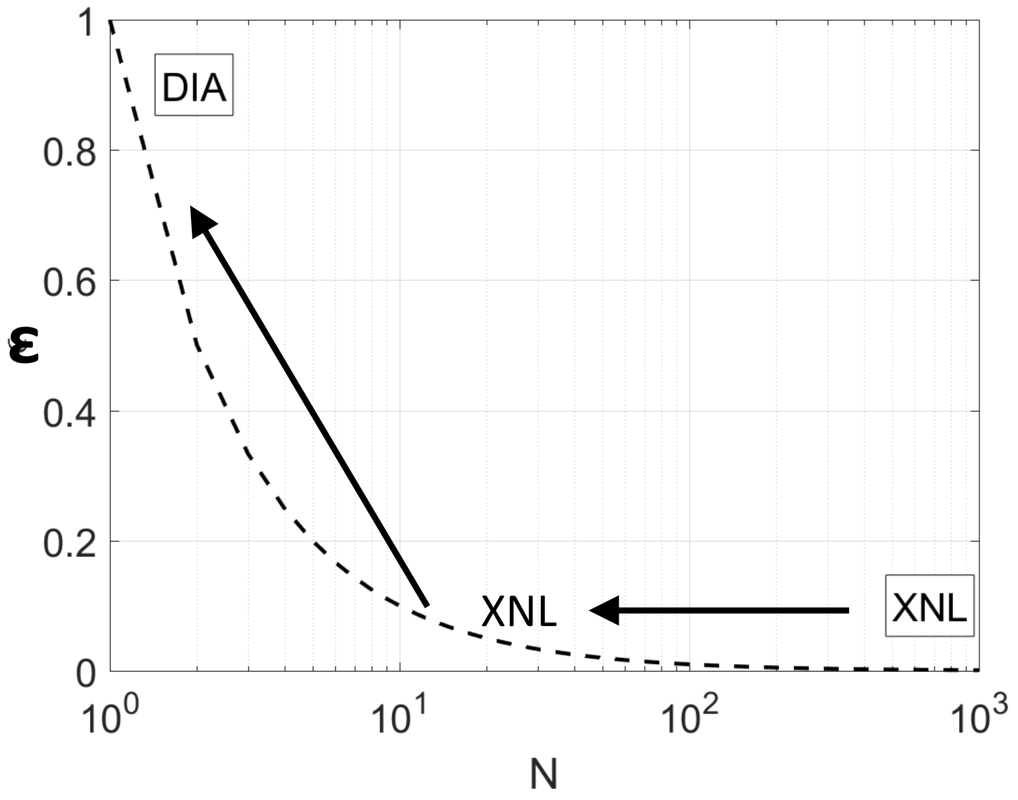


Consequences of scalable method and quantification of model performance

Seek balance between required accuracy and computational workload

Retuning might be needed

$$\varepsilon_{T13} \Rightarrow \varepsilon_{Snl4} \Rightarrow \varepsilon_{\text{model}} \begin{cases} H_{m0}, T_p, T_{m01}, \theta_m, \sigma \\ E(f), E(\theta) \\ E(f,\theta) \end{cases}$$



Conclusions

- Quadruplet interactions play an **essential** role in wave evolution
- Do **not** focus only on having an accurate computational method
- Seek **balance** between accuracy of Snl4 term and operational requirements
- A **scalable** method has been developed for computing Snl4
- **Objective** quantification of computational workload N
- Error ε can be quantified at **different** levels
- Balance between N and ε depends on application and the **quantification of model performance**
- Modify i**mplementation** of XNLINIT and XNLMAIN in operational models like SWAN and Wavewatch III

Thank you